ESTIMATION OF VERTEX FROM THE TOTAL TIME OF PLIGHT TO GRAZE AND MODIFICATION OF SLADEN'S FORMULA

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The limitations of the applicability of Sladen's formula for estimating vertex heights are discussed and the errors involved in its use are shown. An improved formula for rapid calculations of vortex height from the time of flight to graze is given. A better method for the estimation of vertex height by extrapolation or interpolation, only assuming that the vertex heights at two times of flight are known, is also given.

In the Land Service Range Tables, information regarding vertex height of the trajectories for various ranges is not provided, but is estimated by Sladen's formula

$$Y_0 = 4T^2 \tag{1}$$

where Y_0 is vertex height in feet and T is the total time of flight to graze (hereafter called the time of flight) in seconds. The formula is true for vacuum trajectories ($g=32$ ft/sec^2.) and it gives fairly good results in calculating vertex heights in air also'. However, the following examples show that the vertex heights so obtained are 10—15 per cent less than the actual values obtained by numerical integration of the equations of motion of the projectile in air. Sladen's formula, therefore, does not give good approximation to the vertex heights of trajectories in air, as claimed, in most of the cases and needs modification.

Vertex height and time of flight

The vertex height Y_0 in feet can be expressed as a power function. of the time of flight T in seconds

$$Y_0 = AT^{\alpha} \tag{2}$$

where A and α are constants. In vacno a =2 and A=4 ($g=32$ ft/sec^2) which is the form in which the Sladen's formula is expressed. In air, the value of A and α depend upon such parameters as Muzzle velocity (M. V.), Standard Ballistic Coefficient and Angle of Projection.

Assuming that the values of A and α remain constant within two trajectories with closer times of flight, A and α can be determined from the equations

$$\log A = \frac{\log T_1 \times \log Y_2 - \log T_2 \times \log Y_1}{\log T_1 - \log T_2}$$

and

$$\alpha = \frac{\log Y_1 - \log Y_2}{\log T_1 - \log T_2}$$

Where T_1, T_2 and Y_1, Y_2 are the times of flight and vertex heights respectively of the two trajectories.

Knowing A and a the vertex height of any other trajectory with time of flight T lying between T_1 and T_2 can be easily estimated.

Improved version of Sladen's formula

The main difficulty in the application of equation (2) is that it necessitates accurate knowledge of A and a and the extent of their variation with respect to muzzle velocity standard ballistic coefficient and angle of projection.

The calculations being time consuming and difficult to be used under active service conditions, it is necessary to establish an improved version of Sladen's formula of the form

$$Y_0 = BT^2 \tag{3}$$

to cover all combinations of MV, calibre and time of flight. B is a constant and its value depends upon the zone in which lies any particular coinbination of the three factors.

Errors involved in using Sladen's formula

In Tables 1—10, column 2 gives the vertex height obtained from the numerical integration of the equations of motion; column 4, the vertex height calculated by using tho Sladen's formula and column 5, the discrepancies to the nearest ten feet. From these tables it is quite evident that the vertex height cannot always be accurately assessed by Sladen's formula and certain limitations, not specified so far, are to be inposed on its use.

TABLES 1—10

ACTUAL VERTEX HEIGHTS AND VERTEX HEIGHTS ESTIMATED BY SLADEN'S FORMULA AND IMPROVED SLADEN'S FORMULA ALONG WITH THE DISCREPANCIES INVOLVED IN THEIR USE

TABLE 1

Value of parameters	Actual Vertex Height (Y_0) ft.	Time of flight T secs.	Vertex Height by Sladen's formula (Y) ft.	Y_0—(Ys) to the nearest 10 feet ft.	Vertex Height by improved Sladen's Formula (Y_1s) ft.	Y_0—(Y_1s) to the nearest 10 feet ft.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
$MV = 728$ ft sec.	210	7·1	202	10	..	..
Calibre = 2·992″	380	9·7	376	0	..	..
$B = 4 \cdot 04$	660	12·7	645	10	..	..
	1,040	16·1	1,037	0	..	..
	1,610	20.0	1,600	10	..	..
	2,730	26·1	2,725	0	..	..
	5.170	35·8	5,127	40	5,178	−10
	5,960	38·3	5,867	90	5,929	+30

TABLE 2

(1)	(2)	(3)	(4)	(5)	(6)	(7)
MV=1275 ft./sec.	25	2·48	25	0	..	..
Calibre=4″	110	5·20	108	0	..	..
B=4·07	270	8·16	266	0	..	..
	530	11·37	517	10	526	0
	900	14·88	886	10	901	0
	1,430	18·77	409	20	1,434	0
	1,780	20·90	1,747	30	1,778	0
	2,200	23·20	2,153	50	2,191	+10
	2,700	25·70	2,642	60	2,688	+10
	3,300	28·45	3,238	60	3,294	+10
	4,060	31·57	3,987	70	4,056	0
	5,100	35·41	5,015	80	5,103	0
	7,100	41·77	6,979	120	7,101	0

TABLE 3

(1)	(2)	(3)	(4)	(5)	(6)	(7)
M.V.=2000 ft./sec.	110	5·07	103	10	..	..
Calibre=6″	200	7·06	199	0	..	..
B=4·22	350	9·23	341	10	..	..
	540	11·61	539	0		
	810	14·19	805	0	..	..
	1170	16·96	1150	20		
	1620	19·91	1586	30	..	..
	1890	21·47	1844	50	1946	—60
	2200	23·08	2131	70	2248	—50
	2550	24·73	2446	100	2581	—30
	2930	26·43	2794	140	2948	—20
	3350	28·18	3176	170	3351	0
	3810	29·99	3598	210	3795	+10
	4310	31·87	4063	250	4286	+20
	4860	33·83	4578	280	4830	+30
	5460	35·87	5147	310	5430	+30
	6130	38·00	5776	350	6094	+40
	6880	40·25	6480	400	6837	+40
	7720	42·66	7280	440	7680	+40
	8680	45·29	8205	470	8656	+20
	9800	48·24	9208	490	9820	+20
	11240	51·69	10687	550	11275	—40
	13330	56·16	12616	710	13310	+20

TABLE 4

(1)	(2)	(3)	(4)	(5)	(6)	(7)
MV=1875 ft./sec.	20	2·01	16	0	..	
Calibre=1·849″	50	3·25	42	10	..	
B=4·30	90	4·64	86	0	..	
	160	6·18	153	10	..	
	260	7·87	248	10	..	..
	410	9·71	377	30	405	0
	590	11·70	548	40	589	0
	820	13·84	766	50	824	0
	1110	16·16	1045	70	1123	—10
	1470	18·68	1396	70	1500	—30
	1960	21·42	1835	120	1973	—10
	2630	24·52	2405	230	2585	+40

TABLE 5

(1)	(2)	(3)	(4)	(5)	(6)	(7)
MV=2600 ft./sec.	60	3·99	64	0	..	..
Calibre=4″	130	5·61	126	0	..	..
B=4·33	220	7·41	220	0	..	..
	350	9·39	353	0	..	..
	530	11·54	533	0	..	..
	810	13·92	775	30	..	..
	1170	16·57	1098	70	1189	—20
	1620	19·46	1515	100	1640	—20
	1880	20·98	1761	120	1906	—30
	2170	22·55	2034	140	2202	—30
	2490	24·17	2337	150	2530	—40
	2850	25·84	2671	180	2891	—40
	3260	27·56	3038	220	3289	—30
	3720	29·33	3441	280	3725	0
	4220	31·16	3884	340	4204	+20
	4770	33·05	4369	400	4730	+40
	5370	35·02	4906	460	5310	+60
	6020	37·05	5491	530	5944	+80
	6730	39·17	6137	590	6643	+90
	7500	41·38	6849	650	7414	+90
	8350	43·70	7639	710	8269	+80
	9290	46·14	8516	770	9218	+70
	10370	48·73	9498	870	10282	+90
	11630	51·57	10638	990	11515	+110
	13130	54·86	12038	1090	13032	+100
	15070	59·14	13990	1080	15144	—70
	18450	65·28	17046	1400	18452	0

TABLE 6

(1)	(2)	(3)	(4)	(5)	(6)	(7)
MV=2350 ft./esc.	30	2·74	30	0	..	..
Calibre=4·5″	70	4·27	73	0	..	..
B=4·27	140	5·92	140	0	..	..
	230	7·71	238	—10	..	..
	370	9·65	373	0	..	..
	560	11·78	555	0	..	..
	810	14·11	796	10	..	..
	1130	16·66	1110	20	..	..
	1530	19·44	1512	20	..	..
	1760	20·89	1746	10	..	..
	2020	22·37	2002	20	..	..
	2320	23·88	2281	40	..	..
	2660	25·42	2585	80	2759	—100
	3030	27·00	2916	110	3113	—80
	3440	28·62	3276	160	3498	—60
	3880	30·29	3670	210	3918	—40

TABLE 6

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	4360	32·01	4099	260	4375	—20
	4890	33·79	4567	320	4875	+10
	5460	35·62	5075	380	5418	+40
	6070	37·51	5628	440	6008	+60
	6730	39·47	6232	500	6652	+80
	7440	41·51	6892	550	7358	+80
	8220	43·65	7621	600	8136	+80
	9080	45·90	8427	650	8996	+80
	10030	48·28	9324	710	9953	+80
	11100	50·83	10335	760	11032	+70
	12330	53·64	11509	820	12286	+40
	13820	56·85	12928	890	13800	+20
	15750	60·83	14801	950	15800	+50

TABLE 7

(1)	(2)	(3)	(4)	(5)	(6)	(7)
MV=2350 ft./sec.	30	2·75	30	0		
Calibre=4·5″	70	4·30	74	0		
B=4·26	140	5·97	143	0		..
	250	7·79	243	10		..
	400	9·76	381	20		..
	590	11·90	566	20		
	800	14·22	809	—10		
	1080	16·76	1124	—40		
	1480	19·49	1519	—40		
	1730	20·93	1752	—20		
	2020	22·41	2009	10		
	2350	23·94	2292	60	..	..
	2710	25·53	2607	100	2776	—70
	3100	27·18	2955	140	3147	—50
	3520	28·89	3339	180	3556	—40
	3980	30·65	3758	220	4002	—20
	4480	32·45	4212	260	4486	—10
	5020	34·30	4706	310	5012	+10
	5610	36·22	5248	360	5589	+20
	6260	38·21	5840	420	6220	+40
	6970	40·29	6493	480	6915	+50
	7750	42·47	7215	530	7684	+70
	8610	44·76	8014	600	8535	+70
	9560	47·19	8908	650	9487	+70
	10630	49·80	9920	710	10565	+70
	11870	52·68	11101	770	11822	+50
	13360	56·01	12548	810	13364	0
	15330	60·08	14438	890	15377	—50
	18830	66·65	17769	1060	18924	—90

TABLE 8

(1)	(2)	(3)	(4)	(5)	(6)	(7)
MV=2700 ft./sec.	60	3·68	54	10	..	..
Calibre=6″	110	5·08	103	10	..	..
B=4·33	180	6·57	173	10	..	..
	270	8·18	268	0	..	..
	400	9·96	392	10	..	..
	560	11·74	551	10	..	..
	750	13·76	757	—10	..	..
	990	I.?·94	1016	—30	..	..
	1320	18·28	1337	—20	..	..
	1740	20.81	1732	10	..	..
	2000	22·14	1961	40	..	..
	2290	23·51	2211	80	2393	—100
	2600	24·92	2484	120	2689	—90
	2940	26·37	2782	160	3011	—70
	3310	27·87	3107	200	3363	—50
	3710	29·42	3462	250	3748	—40
	4140	31·01	3846	290	4164	—20
	4610	32·64	4261	350	4613	0
	5110	34·31	4709	400	5097	+10
	5640	36·02	5190	450	5618	+20
	6200	37·77	5706	490	6177	+20
	6790	39·56	6260	530	6776	+10
	7420	41·39	6853	570	7418	0
	8100	43·27	7489	610	8107	+10
	8840	45·19	8169	670	8842	0
	9630	47·16	8896	730	9630	0
	10480	49·19	9679	800	10477	0
	11400	51·29	10523	880	11391	+10
	12400	53·48	11440	960	12384	+20
	13480	55·79	12450	1030	13477	0
	14650	58·25	13572	1080	14692	—40
	15930	60·89	14830	1100	16054	—120

TABLE 9

(1)	(2)	(3)	(4)	(5)	(6)	(7)
MV=2300 ft./sec.	11	1·65	11	0	..	..
Calibre=1·563″	33	2·80	31	0	..	..
B=4·36	70	4·18	70	0	..	..
	140	5·72	131	10	143	0
	240	7·43	221	20	241	0
	380	9·29	345	40	376	0

TABLE 10

(1)	(2)	(3)	(4)	(5)	(6)	(7)
MV=3180 ft./sec.	20	2·23	20	0	..	..
Calibre=1·574″	56	3·70	55	1	..	..
B=4·53	124	5·52	122	2	..	..
	183	6·59	174	9	..	..
	260	7·80	243	17	..	..
	362	9·15	335	27	379	—17
	491	10·63	452	39	512	—21
	651	12·22	497	54	676	—25
	846	13·8	762	84	863	—17
	1083	15·4	949	134	1074	+19
	1368	17·4	1211	157	1372	—4
	1701	19·4	1505	196	1705	—4
	2095	21·5	1849	246	2094	+1
	2591	23·8	2266	325	2566	+25
	3116	26·2	2746	370	3110	+6
	3785	28·7	3295	490	3731	+54
	4595	31·7	4020	575	4552	+43
	5578	35·1	4928	650	5581	—3
	6749	38·6	5960	789	6750	—1
	8161	42·7	7293	868	8260	—99
	10173	47·9	9178	995	10394	—221

NOTE—111 table 10 the figures are to the nearest foot.

Limitations of Sladen's formula

From Tables 1—10, it is evident that for estimating the vertex heights of trajectories in air, Sladen's formula can be usefully applied only for subsonic muzzle velocities. For high velocity projectiles this formula only gives a reasonably accurate estimate of the vertex height for short times of flight. For higher times of flight Sladen's formula is not at all *&a-ble. This limitation necessitates imposition of certsin practical restrictions' on the use of Sladen's formula thereby narrowing its scope of application.

Maximum limit of the time of flight up to which Sladen's formula can be employed usefully, depends upon the velocity and calibre of the projectile. The details are given in Table 11.

TABLE 11

LIMITS OF PARAMETERS FOR THE APPLICATION OF SLADEN'S FORMULA

MV of the projectile lying in the velocity zone of (ft./sec.)	Calibre of the projectile (inch)	Limits of times of flight when Sladen's formula can be applied (sec)
700—1000	All Calibres	25—35
1000—1500	Over 3″	20—25
	Under 3″	15—20
1500—2000	Over 3″	15—20
	Under 3″	8—10
2000—3500	Over 3″	16—20
	Under 3″	5—8

It would be seen from the above that the scope of application of Sladen's formula for estimating the vertex height of trajectories with a reasonable degree of accuracy is quite limited.

Improved Sladen's formula and value of constant 'B'

If in a combination of three variables *viz.*, time of flight, velocity and calibre of projectile, the value of any one variable falls out of the limits given in Table 11, the vertex height cannot be accurately determined by Sladen's formula and the use must be made of improved Sladen's formula $Y_c = BT^2$. In Tables 1—10, column (6) gives the vertex heights obtained by improved version of Sladen's formula and column (7) gives discrepancies (to the nearest ten feet) between the actual vertex heights and those estimated by employing improved Sladen's formula. Comparison of values given in columns (4) and (6) reveal the degree of improvement obtained by using improved version of Sladen's formula.

The values of '*B*' for various zones of variables which give a fairly accurate estimate of vertex heights are given in Table 12.

TABLE 12

VALUE OF '*B*' AND IST VARIATION

Velocity zone in which *MV* of the projectile lies (ft/sec)	Calibre of the projectile (inch)	Value of constant 'B'	One inch increase, decrease in calibre of projectile change '*B*' by	+100 ft/sec. in *MV* of projectile increase 'B' by
1000—1500	Over 3"	4·05 to 4·10	—0·01	+0·01
	Under 3"	4·10 to 4·15	+0·01	+0·01
1500—2000	Over 3"	4.20 to 4·25	—0.01	+0·01
	Under 3"	4·25 to 4·30	+0·01	+0·01
2000—3500	Over 3"	4·25 to 4·40	—0·01	+0·015
	Under 3"	4·35 to 4.60	+0·01	+0·015

Table 12 shows that the value of 'B' always lies in between 4·0 to 4·6 and it increases with increase of muzzle velocity and decreases with an increase in calibre of the projectile. Generally the value of '*B*' varies in accordance with the variations given in columns 4 and 5 of the Table 12. The values of 'B' given on the top of each table and used for calculating the vertex heights tabulated in column 5 of the Tables 1—10 are readily obtained from Table 12.

CONCLUSION

From the foregoing discussions, it can easily be seen that the vertex heights by Sladen's formula can only be accurately determined within the limits specified in Table 11. In other words, the value of the constant *B* of equation (3) ran be taken as 4 within the limits of Table 11. If any of the three parameters *viz.*, muzzle velocity, calibre and time of flight is beyond the limits specified in the Table 11, the value of *B* should, for better accuracy be obtained from Table 12 for calculating the vertex heights with the help of the improved version of Sladen's formula as given in equation (3).

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